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Improved Lower Bounds on the
Length of Davenport-Schinzel Sequences

by

Micha Sharir

Technical Report No. 204
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Improved Lower Bounds on the Length of Davenport-Schinzel Sequences

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ABSTRACT

We derive lower bounds on the maximal length $\lambda_s(n)$ of (n, s) Davenport Schinzel sequences. These bounds have the form $\lambda_{2s+1}(n) = \Omega(n\alpha^s(n))$, where $\alpha(n)$ is the extremely slowly growing functional inverse of the Ackermann function. These bounds extend the nonlinear lower bound $\lambda_3(n) = \Omega(n\alpha(n))$ due to Hart and Sharir [HS], and are obtained by an inductive construction based upon the construction given in [HS].

1. Introduction

In this paper we obtain improved nonlinear lower bounds on the maximal length $\lambda_s(n)$ of an (n, s) *Davenport-Schinzel sequence* (a $DS(n, s)$ sequence in short). Such a sequence $U = (u_1, \dots, u_m)$ is defined to be a sequence composed of n distinct symbols which satisfies the following two conditions:

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- (i) $u_i \neq u_{i+1}$ for all i ;
- (ii) U does not contain any alternating subsequence of two (distinct) symbols whose length is $s+2$; that is, there do not exist $s+2$ indices $i_1 < i_2 < \dots < i_{s+2}$ such that

$$u_{i_1} = u_{i_3} = u_{i_5} = \dots = a ,$$

$$u_{i_2} = u_{i_4} = u_{i_6} = \dots = b ,$$

and $a \neq b$.

These sequences provide a combinatorial characterization of the lower envelope of n continuous functions, each pair of which intersect in at most s points. In fact $\lambda_s(n)$ is equal to the maximal number of connected graph portions of such functions which compose their lower envelope (cf. [DS], [Da], [At]). Analysis of these sequences has thus numerous applications in combinatorial and computational geometry, for which see [At], [HS], [OSY], [BS], [LS].

The major known estimates for $\lambda_s(n)$ are:

- (I) $\lambda_1(n) = n$; $\lambda_2(n) = 2n - 1$ (trivial; cf. e.g. [At]).
- (II) $\lambda_3(n) = \Theta(n\alpha(n))$ (Hart and Sharir [HS]), where $\alpha(n)$ is the functional inverse of Ackermann's function.
- (III) $\lambda_s(n) = O(n\alpha(n)^{O(\alpha(n)^{-3})})$ (Sharir [Sh]). This is an improvement of the upper bound $\lambda_s(n) \leq C_s n \log^* n$ obtained by Szemerédi [Sz], where C_s is a positive constant depending on s , and where $\log^* n$ is the height of the smallest exponential tower $2^{2^{\dots^2}}$ which is $\geq n$.

The only previously known nontrivial lower bound on $\lambda_s(n)$ was that obtained in [HS]. In this paper we extend the technique of [HS] to obtain the lower bounds

$$\lambda_{2s+1}(n) = \Omega(n\alpha^s(n))$$

for each $s \geq 1$ and each $n \geq 1$, where the constant of proportionality depends on s .

2. A modified inductive construction of nonlinear DS sequences.

Our construction of $DS(n, 2s+1)$ sequences of length $\Omega(n\alpha^s(n))$ proceeds by induction on s based on the construction of nonlinear $DS(n, 3)$ sequences given in [HS, Section 6]. We assume here familiarity of the reader with the (fairly involved) construction of [HS], but we begin by briefly recalling some of its important features.

Given a tree T , a *generalized path compression* (GPC in short) f on T is a sequence (x_1, \dots, x_k) of nodes of T , all lying along the same path of T from some leaf to the root, and ordered so that for each i the node x_i is a (not necessarily direct) descendant of x_{i+1} ; the compression f amounts to modifying T so that each of x_1, \dots, x_{k-1} becomes a direct child of x_k , and all other parent-child relationships in T remain unchanged. f is said to have length $k-1$, and to *pass through* the nodes x_1, \dots, x_{k-1} ; x_1 is called the *starting node* of f . An (admissible) *path compression scheme* F on T is a sequence of GPC's (f_1, \dots, f_t) on T such that (i) the starting nodes of the f_i 's are arranged in tree postorder, and (ii) the nodes of f_i still lie along a single path of the tree after it has been compressed by f_1, \dots, f_{i-1} in this

order.

The construction of [HS] yields, for each pair of integers k, m , an admissible path compression scheme on a balanced mostly binary tree $T(k, m)$ such that exactly one generalized path compression (GPC) starts at each leaf of $T(k, m)$, and such that each of these GPC's has length k . The tree $T(k, m)$ is obtained by taking a complete binary tree of depth $B_k(m)$, and by replacing each of its leaves by a copy of a *base tree* $T = T(1, m)$ consisting of a root and m leaves, all of which are direct children of the root. The functions $B_k(m)$ are a collection of rapidly growing functions defined inductively as follows:

$$\begin{aligned} B_1(m) &= 0, & m \geq 0, \\ B_k(0) &= 1, & k \geq 2, \\ B_k(m) &= B_k(m-1) + B_{k-1}(2^{B_k(m-1)}), & k \geq 2, m \geq 1. \end{aligned}$$

Recall also that in transforming such a path compression scheme to a *DS* sequence $S(k, m)$, the nodes of $T(k, m)$ become *chains* in S (ordered in their tree postorder), and the GPC's become the symbols of which $S(k, m)$ is composed. Each chain c consists of all the symbols (GPC's) which pass through the corresponding tree node n (so n is not the last node of any of them), arranged in *decreasing* postorder of their starting leaves, and $S(k, m)$ is the concatenation of all these chains (in their postorder). Thus $S(k, m)$ is a sequence composed of

$$N(k, m) = m \cdot 2^{B_k(m)} \quad (1)$$

symbols spread over about that many chains. (Note that $S(k, m)$ may not be a proper *DS* sequence, because it can contain pairs of equal adjacent elements.

However, as noted in [HS], the transformation from this sequence to a proper *DS* sequence is straightforward and does not change the length of the sequence by more than an amount linear in N .)

In what follows it will be convenient to use the functions

$$C_k(m) = 2^{B_k(m)}$$

which thus satisfy the formulas

$$C_1(m) = 1, \quad m \geq 0,$$

$$C_k(0) = 2, \quad k \geq 2,$$

$$C_k(m) = C_k(m-1) \cdot C_{k-1}(C_k(m-1)), \quad k \geq 2, m \geq 1.$$

In particular note that $N(k, m) = mC_k(m)$. Note also that $C_1(1) = 1$, $C_2(2) = 2$, $C_3(3) = 16$, and $C_4(4) > 2^{2^{21}}$, as can be easily checked.

Let γ_k be the functional inverse of C_k , i.e.

$$\gamma_k(m) = \min \{j : C_k(j) \geq m\}$$

Similariy, let γ be the functional inverse of the diagonalization $C(k) = C_k(k)$.

The analysis in Section 6 of [HS] implies

$$A_{k-1}(m) \leq C_k(m) \leq A_k(m+3)$$

for $k \geq 4$, $m \geq 1$. where $A_k(m)$ are the standard Ackermann's functions (cf. [HS] for more detail). These inequalities easily imply (for small values of m this should be checked directly)

$$\gamma(m) - 2 \leq \alpha(m) \leq \gamma(m) + 3 \tag{2}$$

so that these two functional inverses are almost equal. In particular the results of [HS] imply

$$\lambda(m) \geq cm\gamma(m)$$

for some positive constant c and each $m \geq 1$.

Next let $s = 2q+1$ for some $q > 1$. Transform $S(k,m)$ into another longer sequence $U_s(k,m)$ as follows. Take each chain c in $S(k,m)$; suppose c consists of t (distinct) symbols. Then replace c by a maximal $DS(t,s-2)$ sequence c^* composed from the same t symbols so that the leftmost appearances of these symbols in c^* occur in the same order as their single appearances in c . $U_s(k,m)$ is then defined to be the concatenation of all these expanded chains.

Lemma 1: After erasures of adjacent equal elements, $U_s(k,m)$ becomes a $DS(n,s)$ sequence, where $n = N(k,m)$.

Proof: By double induction on k, m . First observe that if for some k, m the sequence $U_s(k,m)$ contains an alternation $a b a \cdots b a$ of length $s+2 > 5$, then there must exist a chain in $S(k,m)$ which contains both a and b .

Now consider first the case $k = 1$. $T(1,m)$ is just a depth-1 tree consisting of a root and m leaves, and only the leaves become nonempty chains of $S(1,m)$. Moreover, each chain c corresponding to some leaf l contains only one GPC (namely the GPC starting at l), so it has length 1, and thus $c^* = c$. Hence $U_s(1,m) = S(1,m)$ so the claim is obvious in this case.

Consider next the case $k > 1, m = 1$, and suppose that the claim is true for all $k' < k, m' \geq 1$. It is easy to see that, after erasures, $U_s(k,1)$ becomes equal to $U_s(k-1,2)$, so the claim follows in this case by induction hypothesis for $k-1$.

Finally consider the case $k > 1, m > 1$, and suppose the claim is true for all $k' < k, m' \geq 1$, and for $k' = k, m' < m$. The inductive step in [HS] first

constructs $T(k, m-1)$ and an associated path compression scheme on it by temporarily ignoring the last leaf in each base-tree T . Then it constructs $T(k-1, C_k(m-1))$ and an associated compression scheme by using as a base tree the tree T^* obtained from $T(k, m-1)$ by discarding all nodes other than its root and the $C_k(m-1)$ roots of the copies of the base tree T in $T(k, m-1)$. Finally, the tree $T(k-1, C_k(m-1))$ and the $C_{k-1}(C_k(m-1))$ copies of $T(k, m-1)$, together with their associated compression schemes, are properly merged together in the manner described in [HS] to yield a compression scheme on a tree $T(k, m)$ having the desired properties.

(More specifically, this merging is performed as follows. $T(k, m)$ is obtained from $T(k-1, C_k(m-1))$ by replacing each copy of its base tree T^* by a copy of $T(k, m-1)$ in which each base tree T has now m leaves. The combined path compression scheme on $T(k, m)$ is defined as follows. For a leaf l which is not the last (i.e. the m -th) leaf of its base tree, use the GPC yielded by the compression scheme associated with the copy of $T(k, m-1)$ containing l . If l is the m -th leaf in its base tree, let f_0 be the GPC starting at the root r of that copy of T , as yielded by the compression scheme associated with $T(k-1, C_k(m-1))$; then define the GPC starting at l to be the concatenation of l with f_0 . It is shown in [HS] that this compression scheme is admissible, and clearly each of its GPC's has length k .)

Suppose $U_s(k, m)$ contains an alternation $a b a \cdots b a$ of (odd) length $s+2$. Suppose first that the GPC's a and b both belong to compression schemes on the "small" trees $T(k, m-1)$. If their schemes are associated with different copies of $T(k, m-1)$ then a and b clearly cannot alternate at all in

$S(k, m)$, and thus also in $U_s(k, m)$. If a and b belong to the scheme associated with the same copy of $T(k, m-1)$, then we obtain a contradiction by induction hypothesis for $k, m-1$. Suppose next that a, b both belong to the scheme associated with the "big" tree $T(k-1, C_k(m-1))$. It is easy to check that in this case the induction hypothesis for $k-1$ rules out such an alternation. Suppose finally that a belongs to a scheme associated with a copy of $T(k, m-1)$ and that b belongs to the scheme associated with $T(k-1, C_k(m-1))$. Then, as observed above, a and b must both pass through the same node of $T(k, m)$. But the only nodes through which both types of GPC can pass are the roots of the base-tree T . Thus a and b are GPC's starting at two leaves of the same copy of T , with b starting at the last (rightmost) leaf in that copy. Let r be (the chain in $S(k, m)$ corresponding to) the root of T . Note that b must be the first element in r , and that, after erasures, its first appearance in r^* must also be its first appearance in U_s (indeed, before erasures, the first appearance of b in U_s is in the singleton chain corresponding to b 's starting leaf, which immediately precedes r in postorder; thus after erasures this first appearance is erased, making the above claim obvious). It is also easy to check that U_s contains one occurrence of a before r^* (namely at the chain corresponding to the starting leaf of a), and that, following r^* , all remaining occurrences of a in U_s must precede all remaining occurrences of b . These facts, together with the fact that r^* is a $DS(m, s-2)$ sequence and $s-2$ is odd, easily yield the desired contradiction.

□

Let us, in a slight abuse of notation, denote by $U_s(k, m)$ also the length of that sequence (before erasures). Then we have

Lemma 2:

$$U_s(1, m) = m, \quad m \geq 1, \quad (3)$$

$$U_s(k, 1) = U_s(k-1, 2) + C_k(1), \quad k \geq 2, \quad (4)$$

$$U_s(k, m) = C_{k-1}(C_k(m-1)) \cdot U_s(k, m-1) + \quad (5)$$

$$C_k(m)(\lambda_{s-2}(m) - \lambda_{s-2}(m-1)) + U_s(k-1, C_k(m-1)), \quad k \geq 2, m \geq 2.$$

Proof: (3) follows from the fact that $S(1, m)$ consists of m chains each composed of a single symbol. Thus in this case $U_s(1, m) = S(1, m)$. (4) follows from the construction, noting that the additional chains in $S(k, 1)$ which are not present in $S(k-1, 2)$ correspond to the leaves of $T(k, 1)$, and are thus each composed of a single symbol. Since the number of such leaves is $C_k(1)$, (4) follows. As for (5), note that the first term on its right-hand side represents the sum of the lengths of all the "small" sequences obtained from the compression schemes on the $C_{k-1}(C_k(m-1))$ copies of $T(k, m-1)$ in $T(k, m)$. The third term represents the length of the sequence obtained from the scheme associated with $T(k-1, C_k(m-1))$, in which we can regard each GPC as starting at the m -th leaf of the corresponding base tree and skipping the root of that tree, obtaining this way the full length of the contribution of $S(k-1, C_k(m-1))$ to $S(k, m)$ (and also the full contribution of $U_s(k-1, C_k(m-1))$ to $U_s(k, m)$), but still missing one occurrence of each such GPC f in the resulting sequence $S(k, m)$, namely its occurrence in the chain corresponding to the root of the base tree containing the starting leaf of f . The second term in the right-hand side of c represents the length of all expanded chains of $U_s(k, m)$ containing GPC's from both a scheme associated

with some $T(k, m-1)$ and the scheme associated with $T(k-1, C_k(m-1))$. Indeed, as noted in the proof of Lemma 1, these "common" chains are precisely those corresponding to the roots of the copies of the base tree. Each such root induces in $T(k, m-1)$ a chain consisting of $m-1$ symbols, so the length of this chain in $U_s(k, m-1)$ is $\lambda_{s-2}(m-1)$. However in $T(k, m)$ this root induces a chain consisting of m symbols, so its new length (in $U_s(k, m)$) is $\lambda_{s-2}(m)$. These observations plainly imply (5). \square

We can simplify the above recurrence formulas by putting $U_s(k, m) = C_k(m) \cdot Z_k^{(s)}(m)$. Then we have

$$Z_1^{(s)}(m) = m, \quad m \geq 1, \quad (6)$$

$$Z_k^{(s)}(1) = \frac{1}{2} Z_{k-1}^{(s)}(2) + 1, \quad k \geq 2, \quad (7)$$

$$\begin{aligned} Z_k^{(s)}(m) &= Z_k^{(s)}(m-1) + \lambda_{s-2}(m) - \lambda_{s-2}(m-1) \\ &\quad + \frac{Z_{k-1}^{(s)}(C_k(m-1))}{C_k(m-1)}, \quad k \geq 2, m \geq 2. \end{aligned} \quad (8)$$

Eq. (8) can be rewritten as

$$Z_k^{(s)}(m) = Z_k^{(s)}(1) + \lambda_{s-2}(m) - 1 + \sum_{j=1}^{m-1} \frac{Z_{k-1}^{(s)}(C_k(j))}{C_k(j)},$$

which, using (7), can be reduced to

$$Z_k^{(s)}(m) = \lambda_{s-2}(m) + \sum_{j=0}^{m-1} \frac{Z_{k-1}^{(s)}(C_k(j))}{C_k(j)}, \quad k \geq 2, m \geq 1. \quad (9)$$

Next we derive from (9) an explicit lower bound on $Z_k^{(s)}(m)$, provided m is sufficiently large. Specifically,

Lemma 3: For each $k \geq 2$ and $m \geq C_k(k)$ a power of 2, we have

$$Z_k^{(s)}(m) \geq (k - \epsilon_k) \lambda_{s-2}(m) \quad (10)$$

where

$$\epsilon_k = \sum_{t=2}^k \frac{t^2-1}{C_t(t)}$$

Proof: For $k = 2$ and an arbitrary m , (6) and (9) easily imply

$$Z_2^{(s)}(m) = \lambda_{s-2}(m) + m \geq \lambda_{s-2}(m) \geq (2 - \epsilon_2)\lambda_{s-2}(m)$$

because $\epsilon_2 = 3/2$ as is easily checked.

Next suppose the claim is true up to $k-1$ for some $k > 2$, and consider the case of k and $m \geq C_k(k)$ a power of 2. Note that if $j \geq \gamma_k(m)$ then $C_k(j) \geq m \geq C_k(k) > C_{k-1}(k-1)$. Furthermore, since by definition $C_k(j)$ is a power of 2, (9) implies

$$\begin{aligned} Z_k^{(s)}(m) &\geq \lambda_{s-2}(m) + \sum_{j=\gamma_k(m)}^{m-1} \frac{Z_{k-1}^{(s)}(C_k(j))}{C_k(j)} \geq \\ &\lambda_{s-2}(m) + \sum_{j=\gamma_k(m)}^{m-1} (k-1-\epsilon_{k-1}) \cdot \frac{\lambda_{s-2}(C_k(j))}{C_k(j)}. \end{aligned}$$

However, it is easily checked that if a divides b then $\frac{\lambda_{s-2}(b)}{b} \geq \frac{\lambda_{s-2}(a)}{a}$.

Indeed, let $b = at$. Then a feasible $DS(b, s-2)$ sequence can be obtained by concatenating t copies of a $DS(a, s-2)$ sequence of maximal length (i.e. of length $\lambda_{s-2}(a)$), composed of pairwise disjoint sets of symbols. This construction clearly proves the asserted inequality. Since in our case $C_k(j) \geq m$ for each $j \geq \gamma_k(m)$, and both $C_k(j)$ and m are powers of 2, we have

$$\begin{aligned} Z_k^{(s)}(m) &\geq \lambda_{s-2}(m) + (m - \gamma_k(m)) \cdot (k-1-\epsilon_{k-1}) \cdot \frac{\lambda_{s-2}(m)}{m} = \\ &\left[(k-\epsilon_{k-1}) - \frac{\gamma_k(m)}{m} (k-1-\epsilon_{k-1}) \right] \lambda_{s-2}(m). \end{aligned}$$

Next we claim that

$$\frac{\gamma_k(m)}{m} \leq \frac{k+1}{C_k(k)}$$

Indeed, note first that this inequality is immediate if $m = C_k(k)$. Assume then $\gamma_k(m) = t \geq k+1$. Then $m > C_k(t-1)$ and

$$\frac{\gamma_k(m)}{m} \leq \frac{t}{C_k(t-1)}$$

The assertion thus follows by induction from the following inequality

$$\frac{C_k(j)}{j+1} < \frac{C_k(j+1)}{j+2}$$

which in turn follows immediately from definition of C , namely $C_k(j+1) = C_k(j) \cdot C_{k-1}(C_k(j))$, and the second factor is clearly at least 2, i.e. greater than $\frac{j+2}{j+1}$.

Continuing with our inequalities, we thus have

$$Z_k^{(s)}(m) \geq \left[(k - \epsilon_{k-1}) - \frac{(k-1)(k+1)}{C_k(k)} \right] \lambda_{s-2}(m) = (k - \epsilon_k) \lambda_{s-2}(m)$$

which thus completes the inductive proof of the lemma. \square

Now it is plain that the sequence ϵ_k converges to some positive limit, which we denote by ϵ . Lemma 3 thus implies

$$Z_k^{(s)}(m) \geq (k - \epsilon) \lambda_{s-2}(m)$$

for m a power of 2 greater than or equal to $C_k(k)$. We can now obtain our main result:

Theorem 4:

$$\lambda_{2s+1}(n) = \Omega(n\alpha^s(n))$$

for each $s, n \geq 1$, where the constant of proportionality depends on s .

Proof: By induction on s . The basis $s = 1$ is due to [HS]. Suppose the claim holds for $s-1$, i.e.

$$\lambda_{2s-1}(n) \geq c_{s-1} n \alpha^{s-1}(n)$$

for each $n \geq 1$. By (2) we can also write

$$\lambda_{2s-1}(n) \geq c'_{s-1} n \gamma^{s-1}(n)$$

for each $n \geq 1$ and for some appropriate positive constant c'_{s-1} . Now Lemma 3 implies

$$U_{2s+1}(k, m) \geq C_k(m) \cdot (k - \epsilon) \cdot m \cdot \frac{\lambda_{2s-1}(m)}{m} \geq c'_{s-1}(k - \epsilon) N(k, m) \gamma^{s-1}(m) .$$

Now choose $m = C_{k+1}(k)$, which clearly satisfies the assumption of the preceding lemma. Then (1) implies that $N(k, m) = C_{k+1}(k+1)$, and

$$\frac{U_{2s+1}(k, m)}{N(k, m)} \geq c'_{s-1}(k - \epsilon) \gamma^{s-1}(m) .$$

But $\gamma(m) = k+1$ because $C_k(k) < C_{k+1}(k) = m \leq C_{k+1}(k+1)$, and $\gamma(N(k, m)) = k+1$ by definition. Hence

$$U_{2s+1}(k, m) \geq c'_{s-1}(k+1)^{s-1}(k - \epsilon) N(k, m) \geq c_s N(k, m) \gamma^s(N(k, m)) .$$

for some appropriate positive constant c_s . From this we can obtain the desired lower bound for $2s+1$ and for *all* $n \geq 1$ using a straightforward argument similar to that in the proof of Corollary 6.2 of [HS]. We leave details of this argument to the reader. This completes the inductive step of the proof, and thus establishes the asserted lower bounds. \square

Remark: In conclusion, we note that there still exists a considerable gap between the new lower bounds on $\lambda_s(n)$ obtained above and the upper bounds obtained in [Sh]. An obvious open problem is to narrow this gap. We believe that the actual value of $\lambda_s(n)$ is closer to the lower bounds obtained here than to the upper bounds in [Sh].

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